Similarity Coloring of DTI Fiber Tracts

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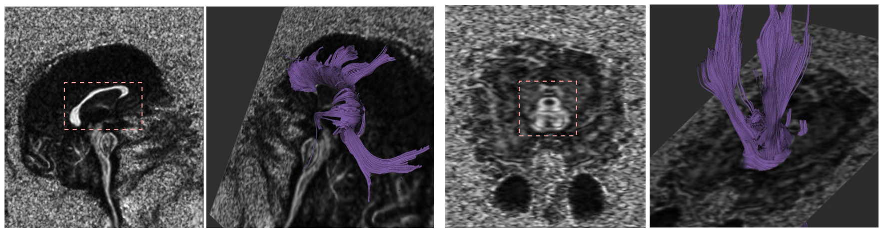
Abstract. We present a coloring method that conveys spatial relations between DTI fiber tracts effectively; similar tracts are assigned to similar colors and different tracts are assigned to different colors in a smooth and continuous manner. To this end, we combine a standard spectral approach with a mass-spring heuristic to embed fiber tracts into a perceptually uniform color space, L*a*b*. We also introduce a new geometric bivariate coloring model, the *flat torus*, that allows finer adjustment of coloring arbitrarily. Results indicate that our method allows a quick evaluation of tract projections and facilitate demarcation of subtle anatomical divisions in fiber tracts.

1 Introduction

Diffusion-Tensor Magnetic Resonance Imaging (DTI) enables the exploration of fibrous tissues such as brain white matter and muscles non-invasively *in-vivo* [1]. It exploits the fact that water in these tissues diffuses at faster rates along fibers than orthogonal to them. Integral curves that represent fiber tracts by showing paths of fastest diffusion are among the most common information derived from DTI volumes [2]. Ability to estimate fiber tracts in vivo is in fact one of the key advantages of DTI over conventional imaging techniques. Integral curves are generated from DTI data by bidirectionally following the principal eigenvector of the underlying diffusion tensor field and often visualized with streamlines or variations of streamlines in 3D. Reflecting the intricacy of the connectivity in the brain, these 3D models obtained from DTI brain data sets are generally visually dense. It is often difficult to ascertain tract projections as well as anatomical and functional structures clearly. Therefore, the ability to see similarities and differences is fundamental to exploring tractography data. In this context, we present a coloring method that conveys spatial relations between tracts effectively: similar tracts are assigned to similar colors and different tracts are assigned to different colors in a smooth and continuous manner. In other words, variation in similarities (and dissimilarities) is represented with proportional variation of colors. To this end, we first compute pair-wise similarities between the tracts then pose the problem of coloring tracts as an optimization problem and solve it.

Contributions: Our main contributions are two fold. First, we present a coloring method where variation in similarities among tracts is reflected with variation in perceptual differences among their colors. Second, we introduce a geometric coloring model, the flat torus, that allows fiber-tract color differences to be adjusted arbitrarily. We also introduce a new fiber-tract similarity measure that uses the complete geometry of fiber pathways while giving higher "importance" to the end points.



(a) (b)

Fig. 1: Two brain fiber bundles superimposed with slices of fractional anisotropy (FA) map. (a) Corpus callosum with the mid-sagittal slice and (b) internal capsule with the mid-coronal slice.

We show example visualizations of the corpus callosum and internal capsule (see Figure 1). The corpus callosum is the largest white matter structure connecting the left and right hemispheres in the brain. The internal capsule runs between the cerebral cortex and the medulla, containing major longitudinal (both ascending and descending) pathway systems, including the corticospinal tract, medial lemniscus, and corticobulbar tract. Both the corpus callosum and internal capsule are targets for clinical and neuroscience research into normal developmental vs. pathological changes in white matter integrity across the lifespan and the functional correlates of those changes.

We have presented some of the ideas in this paper as a poster previously in [3]. Here, we aim to give an extended and more complete view of our work. In the following section we discuss related work. Then, we give details of our coloring method in section 3.1 and present and discuss the results in section 4. We finish the paper by summarizing our work and contributions and by concluding in section 6.

2 Related Work

Mapping data values to colors is a fundamental operation in scientific visualization. Previous work based on empirical studies addressed the problem of generating perceptually effective colormaps [4–7].

Several different geometric models, including lines, planes, cones, cylinders, and B-spline surfaces have been proposed for univariate, bivariate or trivariate color mapping [8, 9]. We extend the earlier models by introducing the flat torus model to give a continuous 2D color mapping that is approximately perceptually uniform and that can be repeated an arbitrary number of times in both directions to increase sensitivity.

Pajevic *et al.* proposed methods to colormap DTI cross-sections according to principal eigenvectors of tensor-valued voxels using different color spaces, including the perceptually uniform CIE L*u*v* color space [10]. The authors point at the potential limitations due to the irregularity of the L*u*v* space. Our flat torus model addresses the limitations due to the shape of the color space by allowing cyclic continuous bivariate mapping.

Integral curves generated from DTI volumes have been visualized generally with streamlines in 3D with different geometric (i.e., hyperstreamlines, streamtubes, etc.) and coloring combinations [11, 12].

In a work that is the closest to ours, Brun *et al.* colored DTI fiber tracts by embedding them in the RGB color space using a non-linear dimensionality-reduction technique [13]. The authors use end-point distances of tracts to define the similarity. Our work differs from this work in 1) the similarity measure that we use, 2) the embedding approach that we take, 3) the perceptual color space that we embed the curves in, and 4) the bivariate coloring model that we introduce.

3 Methods

The goal of our coloring method is to represent the spatial variation among the fiber tracts as perceptual variation among their colors. To this end, we first compute a "distance" matrix quantifying similarities (or dissimilarities) between pairs of tracts. Then, we express the coloring as an optimization problem such that perceptual distances among colors of fiber tracts are proportional to similarities among fiber tracts quantified in the distance matrix. The problem as posed is a distance embedding problem and we approximate the solution using a spectral approach. We refine this global result using a simple mass-spring-based heuristic restricted to local neighborhoods. In the context of our paper, embedding fiber tracts means finding a set of corresponding points in a Euclidean subspace where distances among the points optimally preserve the corresponding distances among the fiber tracts. We use our embedding procedure in two ways. In one, we embed tracts in the three-dimensional color space directly. In the other, we embed them into a plane and color via the flat-torus model by covering the flat-torus surface with the planar embedding.

3.1 Fiber Tracts as Integral Curves

Integral curves are models extracted from DTI volumes to approximate neural fiber tracts in the brain [1]. These integral curves are solutions to the following first-order differential equation: $\frac{dC}{ds} = v_1(C(s))$, where s parametrizes the curve and v_1 is the principal eigenvector of the underlying diffusion tensor field at the point $C(s) = (x(s), y(s), z(s))$. We compute the integral curve $C(s)$ passing through a given seed point $C(0)$ by integrating the above equation for $s > 0$ and $s < 0$ (i.e., both directions from the seed point). We represent integral curves as lists of connected line segments (i.e., polygonal chains).

3.2 Similarities Between Fiber Tracts

We quantify how fiber tracts relate to each other by computing an anatomically motivated pairwise distance measure between them. Our measure tries to capture how much

[1] We use the terms integral curve and fiber tract interchangeably throughout the paper

any given two tracts follow a similar path, while giving more weight to the points closer to tract ends. There have been different distance measures proposed for fiber tracts generated from DTI volumes [14]. In the current work, we modify the chamfer distance measure so that points closer to the ends of the curves have higher weights. Note that our measure does not necessarily satisfy the triangle inequality; therefore, it is not a metric. Given two integral curves $C_i = \{C_i^1, \ldots, C_i^m\}$ and $C_j = \{C_j^1, \ldots, C_j^n\}$ that are represented as polylines with m and n vertices respectively, we first find mean weighted distances d_{ij} and d_{ji}, then determine the maximum of these two distances as the distance D_{ij} between the two curves:

$$d_{ij} = \frac{1}{m} \sum_{k=1}^{m} {\alpha_i}^k \mathrm{dist}(C_i^k, C_j) \tag{1}$$

$$d_{ji} = \frac{1}{n} \sum_{k=1}^{n} {\alpha_j}^k \mathrm{dist}(C_j^k, C_i) \tag{2}$$

$$D_{ij} = D_{ji} = \max(d_{ij}, d_{ji}) \tag{3}$$

The function $\mathrm{dist}(p, C)$ returns the shortest Euclidean distance between the point p and curve C. Also, $\alpha_k = \frac{1}{Z} e^{|k-(m+1)/2)|^2/\sigma^2}$, where the normalizing factor $Z = \sum_{k=1}^{m} e^{|k-(m+1)/2|^2/\sigma^2}$. We set the parameter σ automatically, proportional to L_C, the length of the fiber tract, such that $\sigma = \lambda L_C$, where $\lambda \in (0,1]$. We use $\lambda = 0.5$ for the demonstrations in this paper. We compute distance between each pair of integral and assemble the measures to create a distance matrix. While we believe our distance measure is a good approximation of the notion of similarity in the domain, our coloring method is independent of a particular distance measure.

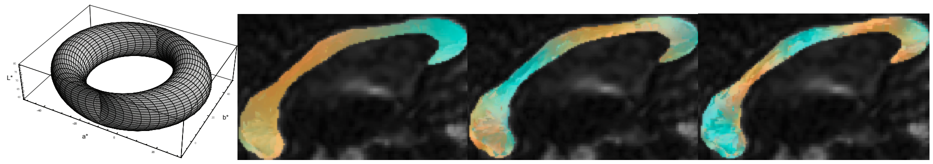
3.3 Embedding Fiber Tracts

In our coloring approach, we aim to reflect the boundaries in distance changes between integral curves as perceptual boundaries of colors. Intuitively, we are looking for a set of color coordinates x in a given color space that optimally preserves relations (as defined in D). Therefore, given a similarity matrix D, we define a cost function $E(x)$ and minimize it.

$$E(x) = \sum_{i<j} (||x_i - x_j|| - D_{ij})$$

Searching for x that minimizes $E(x)$ is indeed a distance embedding problem and we take a spectral approach to approximate it as suggested in [15]. The reason for choosing this particular spectral embedding method is its simplicity and speed. We refine the spectral embedding result using a mass-spring-based heuristic restricted to an ε neighborhood [16].

Perceptually uniform color spaces such as the L*a*b* are natural choices for our method as perceptual differences between colors are encoded as Euclidean distances in these color models. A color space is said to be perceptually uniform if the perceptual difference between any two colors in just-noticeable-difference (JND) units is equal to the Euclidean distance in that color space [17].



(a) (b) (c) (d)

Fig. 2: (a) Bohemian Dome. (b-d) Illustration of the flat torus model adjusting the sensitivity of colormapping by rescaling the planar representation (embedding) of the integral curves and “wrapping” around the flat torus as many times as needed. Pictures, (b-d), show the visualization of the mid-sagittal plane of the corpus callosum with increasing sensitivity of coloring.

We use the embedding procedure above in two different ways to color fiber tracts. In the first, we embed the fiber tracts into the three dimensional L*a*b* color space [2] directly. In the second approach, we use the flat torus as a bivariate coloring model. For this, we first create an embedding in the plane and then map this planar embedding to cover the flat-torus surface arbitrarily many times to adjust the sensitivity of the coloring. Then, we submerse the flat torus in the color space to color the curves.

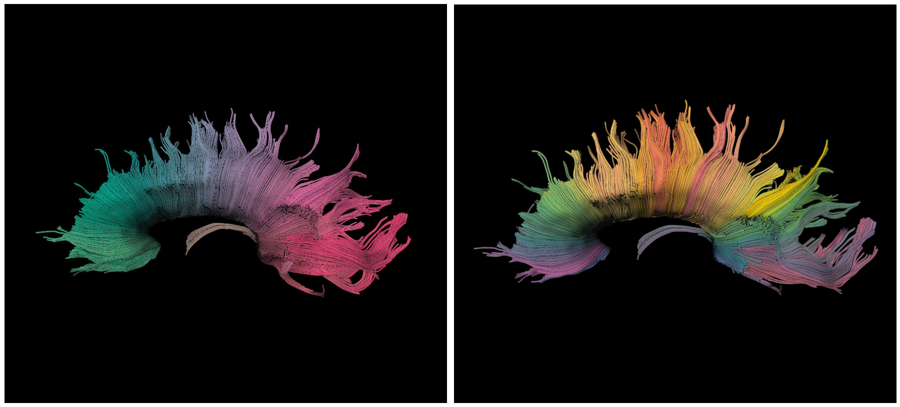
Flat torus We propose the flat torus as a new geometric model for bivariate color mapping. A flat torus in 4-space is a Cartesian product of two circles in R^2. It can be obtained by a mapping $W : R^2 \rightarrow R^4$ such that

$$W(x,y) = (u,v,s,t) = (r_1 \cos x, r_1 \sin x, r_2 \cos y, r_2 \sin y) \tag{4}$$

where r_1 and r_2 are the radii of the circles. The flat torus has zero Gaussian curvature everywhere; therefore, a plane can be wrapped around it without distortion [18]. This can be particularly useful if all four coordinate values are used in a perceptually meaningful way, because wrapping the embedding plane onto the flat torus does not distort the embedded distances. One of the primary advantages of using flat torus is that we can adjust the sensitivity of the color mapping by rescaling the data values (i.e., points on the plane) uniformly and wrapping around the two circles, determined by r_1 and r_2, continuously an arbitrary number of times. This in turn provides a more flexible use of the color bandwidth. Our second coloring approach wraps the planar representation generated by the embedding algorithm onto a flat torus creating 4D (u,v,s,t) coordinates for the plane points. Then we project the flat torus onto a quartic surface called the Bohemian Dome (see Figure 2) centered at $(L_o, a_o + r_1, b_o)$ in a visible portion of the L*a*b* color space as follows:

$$(L*, a*, b*) = (L_o + t, a_o + r_1 + u + s, b_o + v)$$

[2] L*, a*, and b* are the three orthogonal axes of the color space, where L* determines the lightness of the color (ranging from 0 to 100) while a* (ranging green to red) and b* (ranging from blue to yellow) together determine the chroma and hue of the color. See [17] for further details.



(a) (b)

Fig. 3: Corpus callosum colored by (a) embedding the tracts in the L*a*b* color space directly and (b) submersing the flat torus, covered with a planar embedding of the fiber tracts, in the L*a*b* color space. Notice non-local color repetitions in the latter case.

The resulting image shows larger changes in color where neighboring integral curves differ more. Note that this projection is not isometric. It has two lines of self-intersection (where different (x,y) points map to the same colors); it also distorts the angles between the coordinate directions. For the examples shown in this paper, we locate the projected Bohemian Dome in the L*a*b* interval $I = (I_L, I_b, I_c)$, where $I_L = [45,95], I_a = [-50,70], I_b = [-20,70]$, and use $r_1 = 45$, $r_2 = 25$. Note that $L_0 = 70, a_0 = 10$, and $b_0 = 25$.

4 Results

We implemented our distance computation routines in C++ and the embedding procedure in Matlab. Source codes are available online at [19]. We set $\varepsilon = 4.0$ in the spring embedding refinement such that only distances within *epsilon* are iteratively refined. We apply our coloring procedure to the corpus callosum and internal capsule. Figure 3 shows the coloring of the corpus callosum via the flat torus model. Similarly, Figure 4 shows the coloring of the internal capsule using direct embedding into the L*a*b* space.

A common way of coloring fiber tracts in the DTI community is to color-encode the lines determined by the end-points of the tracts (also known as the *end-point vector*, the direction of which does not matter) by mapping the absolute value of the normalized end-point vector to RGB color-triples [20]. Figure 5 demonstrates the advantage of our similarity coloring method over this de-facto standard. The end-point vector coloring gives almost uniform colors (due to mirror symmetries resulted by taking the absolute value of the vector) for both the internal capsule and corpus callosum models, while

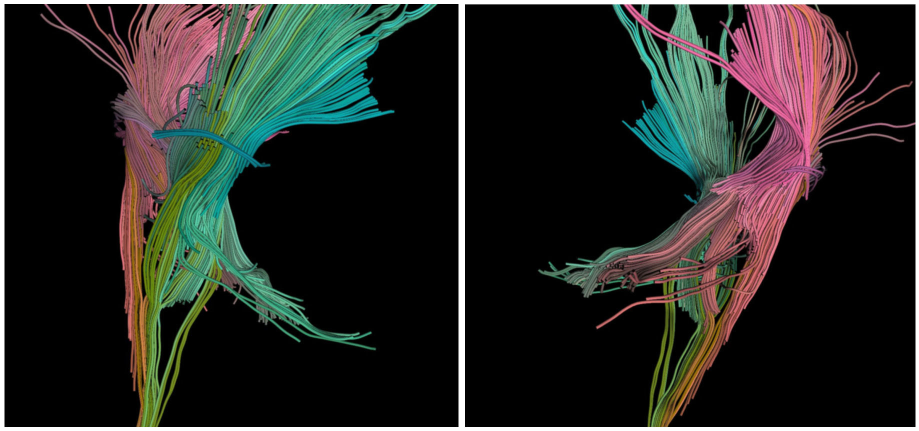
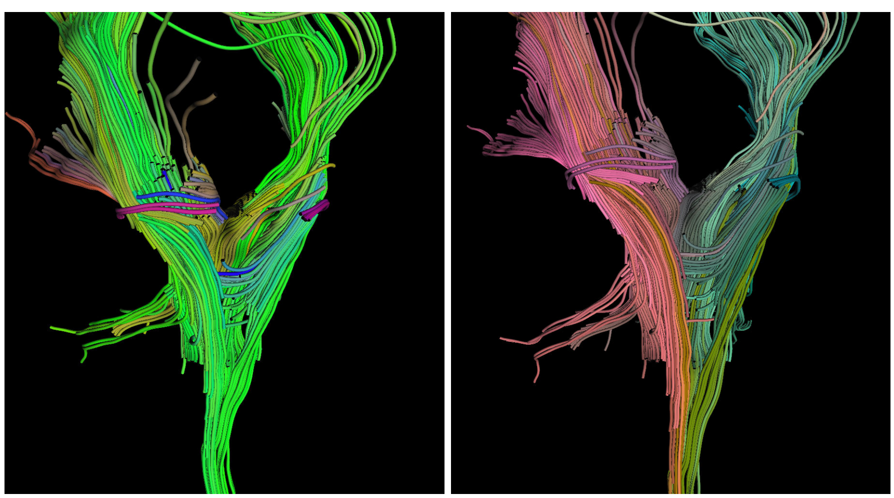


Fig. 4: Two views of the internal capsule colored by embedding the tracts in the L*a*b* space directly.

our method clearly represents the spatial variation among tracts as variation among their colors.



(a) (b)

Fig. 5: Internal capsule colored (a) using the end-point vector coloring and (b) embedding the tracts in the L*a*b* space directly.

5 Discussion

It is important to note that the perceptual uniformity when the flat torus is used is an approximation, because the flat torus cannot be mapped to three space isometrically. Our projection can deemphasize changes in certain regions of the flat torus. There are other projections that may be closer to isometric, although our experimentation with linear and non-linear metric embedding methods such as metric multidimensional scaling (MDS) and isomap [21] using geodesic distances on the flat torus, which are projections of the lines in the lattice space, did not work as well as the Bohemian Dome projection. It also may be possible to add a fourth perceptual dimension like texture to the three color dimensions, removing the need for a projection and preserving the properties of the flat torus. It should be also noted that the perceptually uniformity of the L*a*b* color space is an empirical approximation obtained under certain display settings. Deviation from these settings (e.g., when monitors used are not calibrated) further distorts the approximation of perceptual uniformity [17].

Unless there is an isometry between the “manifold” of fiber tracts and the embedding space (e.g., R^3, R^2), it is not possible to preserve similarities among all pairs of fiber tracts in the embedding space. Our two-step embedding process aims to preserve similarities locally without distorting the global structure too much. However, as the number of fiber tracts increases while ε is kept fixed, it should be expected that both the computational cost and embedding distortion will increase, in general.

The two coloring approaches discussed here have their own merits. While coloring via the flat torus provides a cyclic use of the color space, direct embedding makes better use of the volume of the color space resulting in more saturated colors. Still, for both approaches, initial results indicate that our coloring method can help to quickly ascertain tract projections and find subtler anatomical divisions in fiber tracts.

6 Conclusions

We presented a coloring method that reflects spatial relations between neural fiber tracts effectively, assigning similar tracts to similar colors, and different tracts to different colors, with differences preserved proportionately. We also introduced the flat-torus model that gives the ability to adjust the sensitivity of the color mapping by rescaling the data values (i.e., points on the plane) uniformly and wrapping around the two circles, enabling continuous use of the color “bandwidth.”

We applied our method to the corpus callosum and internal capsule in the brain. Feedback from neuroscientist collaborators suggests that our visualization methods can be useful in identification of smaller caliber anatomically or functionally related white-matter structures, particularly those that are contained within large bundles or fasciculi that project to multiple areas.

While our application interest has been in DTI, the coloring method described here can be applied to curves representing flow, surveillance trajectories, paths, etc. In fact, given a measure of similarity, our method can be used for coloring data entities arising from any field easily.

Acknowledgments

This work has been supported partly by NIH grant number 1R01EB00415501A1 and the Reisman Fellowship of the Brown Institute for Brain Sciences.

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